Transonic Hot-Wire CTA Calibration Methodology

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MOTIVATION

Transonic hot wire anemometry (HWA) is useful for the experimental study of turbulence or entropy generation in high subsonic compressible systems as compressors and fans for turbomachinery applications.
The HWA Instantaneous Measures the Unsteady Transonic Velocity ,Density, Temperature fluctuations, based on universal empirical correlation for heated cylinders in compressible flow

BACKGROUND - Heat transfer over heated wires

- Electrically heated thin wires placed in the flow. The wire, being hotter than the flow, loses heat to the flow. The rate of heat loss from the wire, Q_{tot}, is usually characterized by:

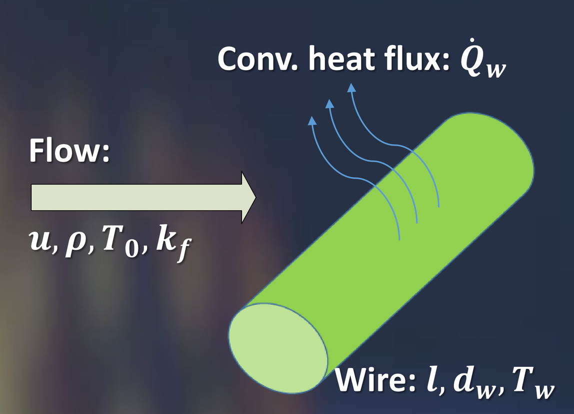
$$\dot{Q}_{tot} = \dot{Q}_{convection} + \dot{Q}_{conduction} + \dot{Q}_{radiation}$$

- the rate of heat transfer from the wire, $\dot{Q}_w = \dot{Q}_{convection}$, is convicted away through the surface area of the wire:

$$\dot{Q}_w = \pi d_w l \cdot h \cdot (T_w - \eta T_0)$$

- The convective heat loss can be rewritten to a non-dimensional form by introducing the use of Nusselt number:

$$Nu = \frac{\dot{Q}_w}{\pi l \cdot k_f (T_w - \eta T_0)}$$



- Nusselt number is defined as:

$$Nu = \frac{h d_w}{k_f} = f(Re, M)$$

- the Reynolds number is:

$$Re = \frac{\rho u d_w}{\mu}$$

- In the equilibrium state, the heat convicts away from the wire and balanced by the power supplied by joule heating:

$$\dot{Q}_w = \frac{E_W^2}{R_w} \qquad R_w = R_{ref}[1 + \alpha_{ref}(T_w - T_{ref})]$$

METHODOLOGY

<u>Empirical Nu Relation for Compressible Flows</u>

$Re - M - Nu$ correlation for infinite [3] :

$$Nu_{corr}(Re_{T_0}, \infty) = \frac{Nu(Re_{T_0}, M)}{\Phi(Re_{T_0}, M)}$$

- Nu_{corr} is independent of M
- Φ represents the Mach dependency

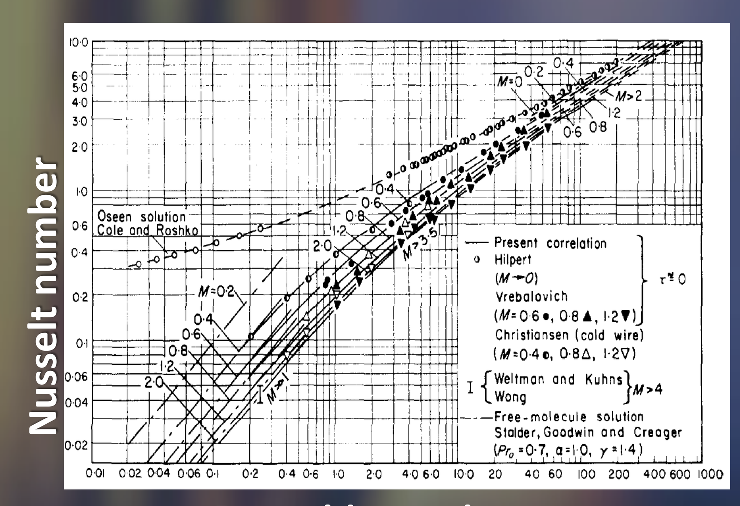


Figure 1. empirical $Re - M - Nu$ correlation [3]

Real life consideration:

- Conduction end losses to prongs
- Geometric imperfections

➔ Φ correctly captures compressibility effects
➔ But $Re - Nu_{corr}$ needs to be determined experimentally

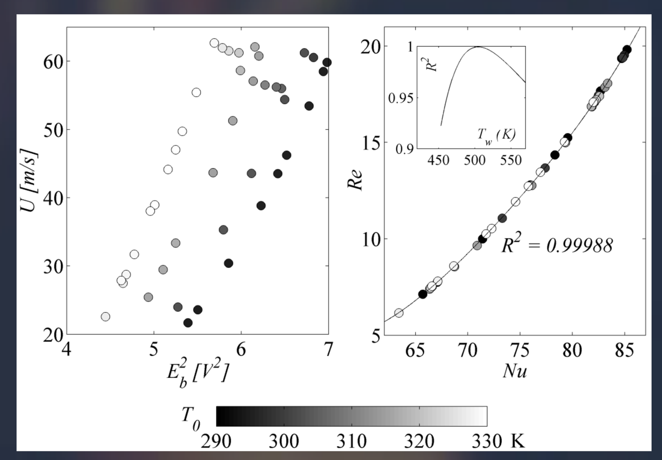


Figure 2. Finding T_w from calibration[4]

<u>Calibration for wire-specific $Re - Nu_{corr}$</u>

Finding effective wire Temperature

- Wire resistance to temperature relation is inaccurate: diameter uncertainties, material impurities, aging
- Recapping heat transfer relation:

$$Nu = \frac{E^2}{\pi l_w k_f R_w} \cdot \frac{1}{(T_w - \eta T_0)}$$

For set R_w, single effective T_w will collapse the data

Compressibility Correction

- Compressibility effect scatters Nu
- Φ correction ➔ single $\mathrm{Re} - Nu_{corr}$ curve

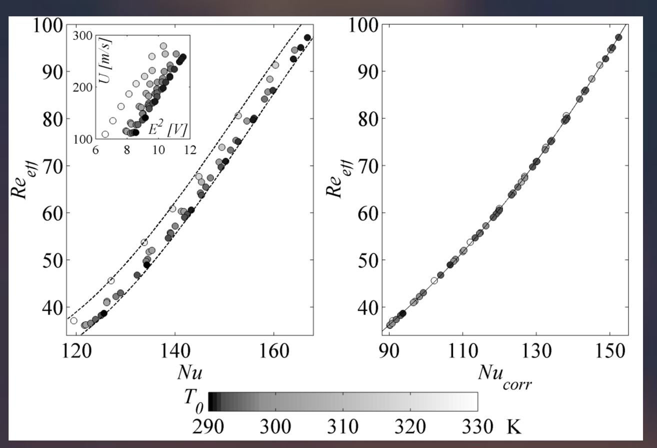


Figure 3. Finding $\mathrm{Re} - Nu_{corr}$ by compressibility correction Φ[4]

DECOUPLING FLOW PERTURBATION QOANTITIES

High speed flows:

Sensitivity relation for perturbations with varying u, ρ, T_0[2]

$$\frac{E'}{\bar{E}} = S_u \cdot \frac{u'}{\bar{u}} + S_\rho \cdot \frac{\rho'}{\bar{\rho}} + S_{T_0} \cdot \frac{T_0'}{\bar{T_0}}$$

$(\cdot)'$perturbation, $\overline{(\cdot)}$ mean quantity

Sensitivity matrix – 3 wire

$$\begin{Bmatrix} \frac{E_1'}{\bar{E_1}} \\ \frac{E_2'}{\bar{E_2}} \\ \frac{E_3'}{\bar{E_3}} \end{Bmatrix} = \begin{bmatrix} S_{u1} & S_{\rho 1} & S_{T_0 1} \\ S_{u2} & S_{\rho 2} & S_{T_0 2} \\ S_{u3} & S_{\rho 3} & S_{T_0 3} \end{bmatrix} \begin{Bmatrix} \frac{u'}{\bar{u}} \\ \frac{\rho'}{\bar{\rho}} \\ \frac{T_0'}{\bar{T_0}} \end{Bmatrix}$$

- External measurement of mean flow quantities needed

➔ Decupling instantaneous flow perturbations achieved

OPTIMIZATION

A value function of two objectives is proposed:

$$Val = max\left(min\left(W_1 \cdot \frac{\left(\kappa(\underline{A}) \left\|\underline{\Delta A}\right\|\right)_{min}}{\kappa(\underline{A}) \left\|\underline{\Delta A}\right\|}, W_2 \cdot \frac{\overline{\left\|\underline{A}\right\|}}{\left\|\underline{A}\right\|_{max}} \right) \right)$$

$\kappa(\underline{A}) = s_{max}/s_{min}$- condition-number of a matrix, in this investigation, Singular Value Decomposition (SVD) is used.

$\left\|\underline{A}\right\|$ - matrix norm and $\left\|\underline{\Delta A}\right\|$- perturbation norm are calculated using the Frobenius norms.

In order to improve invertibility and minimize the amplification of the errors into the decoupled flow perturbations, we are interested in minimizing $\kappa(\underline{A}) \frac{\left\|\underline{\Delta A}\right\|}{\left\|\underline{A}\right\|}$.

The most desirable probe is in the top right corner of the chart, and hence, there are no universally dominant probes; instead they are scattered along a convex Pareto front

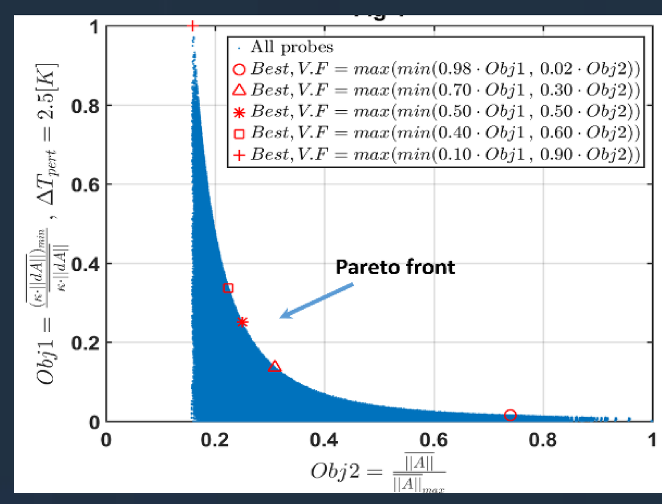


Figure 4. Map of the two objectives for all 4-wire probes combinations

A decoupling quality parameter can be defined:

$$Q_{RMS} = \sqrt{Qual_u^2 + Qual_\rho^2 + Qual_{T_0}^2}$$

To assess the best probe from the pareto front

Value function	d_{wire} $[\mu m]$				T_{wire} $[K]$			
1	5	5	10	10	390	400	390	410
2	5	5	10	10	470	500	470	500
3	5	5	10	10	510	530	520	520
4	5	5	10	10	540	540	530	560
5	5	5	10	10	640	650	640	650

Finally In order to asses the performance of the decoupling a SNR of each quantity is calculated.
Optimization of the wire temperatures is critical, as represented by probe A^*

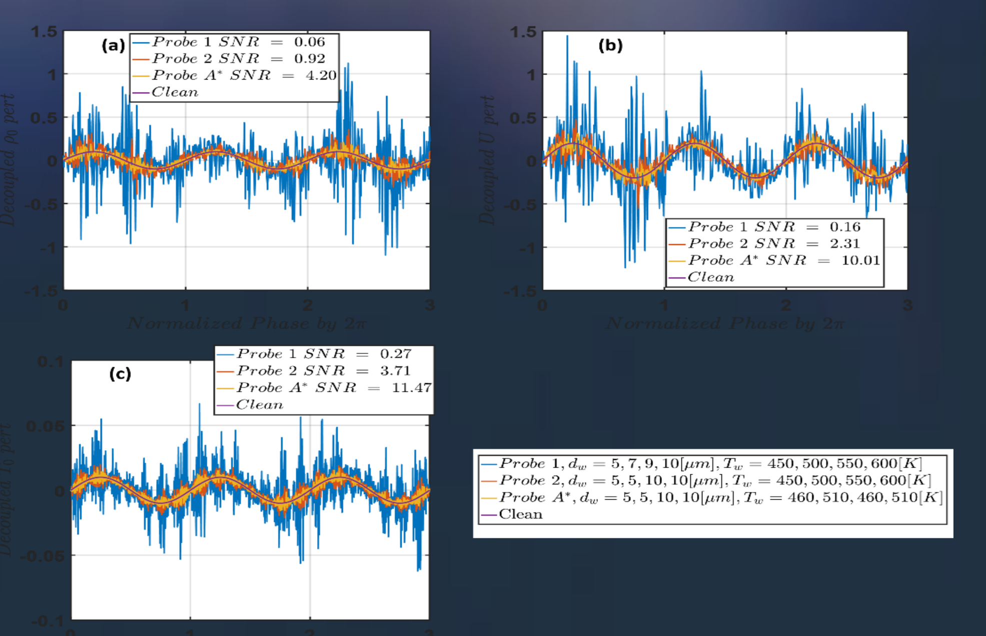
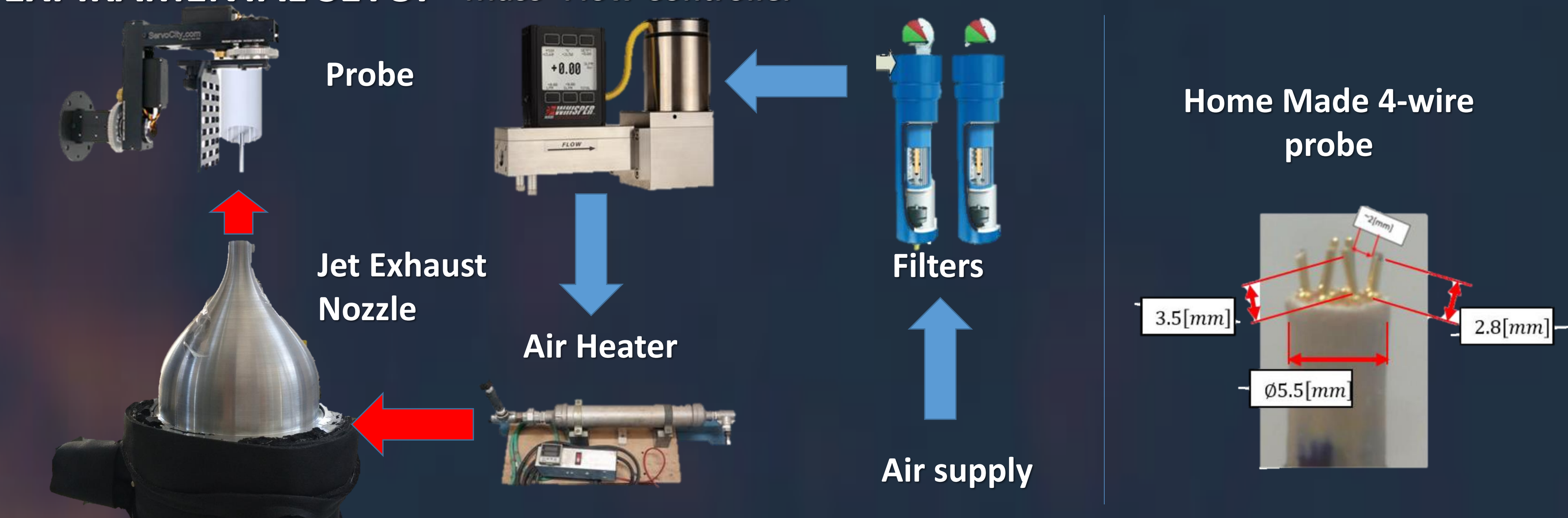


Figure 13 – Selected 4-wire probes decoupling performance comparison for $\bar{M} = 0.9, \overline{T_0} = 350[K], \overline{P_0} = 1.7\ [Atm]$, $Amp_{rel-noise} = 1\%$, $Amp_{DC-noise} = 0.5[mV]$ in terms of (a) decoupled ρ_0 perturbations, (b) decoupled

EXPIRAMENTAL SETUP



FUTURE WORK

- 5 Wire probe construction and calibration
- Decoupling ρ, u, T_0 Fluctuations in a real turbomachinery application

[1] H. Brunn, "Hot-Wire Anemometry," ed: Oxford University Press, New York, 1995.
[2] K. Nagabushana and P. C. Stainback, "Heat transfer from cylinders in subsonic slip flows," 1992.
[3] C. F. Dewey, "A correlation of convective heat transfer and recovery temperature data for cylinders in compressible flow," International Journal of Heat and Mass Transfer, vol. 8, pp. 245-252, 1965.
[4] B. Cukurel, S. Acarer, and T. Arts, "A novel perspective to high-speed cross-hot-wire calibration methodology," Experiments in fluids, vol. 53, pp. 1073-1085, 2012.